ON SOME PROPERTIES OF FLUID FLOW UNDER THE ACTION OF SURFACE TENSION FORCES

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The present paper concerns some problems on the flow of an ideal fluid acted upon by surface tension forces: the possibility of the waves degenerating into solitary waves, the existence of solitary waves of the trough type, the existence of axisymmetrical wave-shaped jets, and the possibility of loss of stability by a jet due to the action of surface tension forces. The study is carried out within the framework of narrow strip asymptotics (see [1 and 2).

1. Long waves in a fluid acted upon by surface tension forces. 1°. The problem of determining steady-state waves on the surface of a heavy fluid acted upon by surface tension forces can be reduced to finding the function ψ harmonic in the strip $0 < y < f(x)$ (Fig.1) and the function $f(x)$ from the conditions

$$\psi = 1 \quad \text{for } y = f(x), \quad \psi = 0 \quad \text{for } y = 0 \tag{1.1}$$

$$\psi_x^2 + \psi_y^2 + 2\nu f - \gamma K = C \quad \text{for } y = f(x) \tag{1.2}$$

Problem (1.1),(1.2) is put in dimensionless form. The depth h of the fluid at a point whose position will be indicated below and the flow rate Q have been chosen as the characteristic parameters. In Expression (1.2) the dimensionless parameter ν, the mean curvature K, and the dimensionless parameter γ characterizing the action of capillary forces are given by the following Formulas:

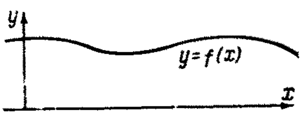

Fig. 1

$$\nu = \frac{gh^3}{Q^2} = \frac{gh}{V}, \qquad K = \frac{f''}{2(1+f'^2)^{3/2}}$$

$$\gamma = \frac{4\alpha h}{Q^2} = \frac{4\alpha}{V^2 h} \qquad \left(V = \frac{Q}{h}\right) \tag{1.3}$$

where V is the characteristic velocity and α is the coefficient of surface tension.

2°. The function ψ harmonic in the strip $0 < y < f(x)$, and its derivatives may be written as (see [1], p.184)

$$\psi(x, y) \sim \frac{y}{f} + \frac{2f'^2 - f''f}{6f} y \left[1 - \frac{y^2}{f^2}\right] + \cdots \tag{1.4}$$

$$v_x = \frac{\partial \psi}{\partial y} \sim \frac{1}{f} - \frac{2f'^2 - f''f}{6f}\left[3\frac{y^2}{f^2} - 1\right] + \cdots, \qquad v_y = -\frac{\partial \psi}{\partial x} \sim \frac{yf'}{f^2} + \cdots$$

Since the boundary $y = f(x)$ is not known in advance, it is necessary that we equip ourselves with some prior estimates of the derivatives of the function $y = f(x)$ (we have in mind the study of long waves and the conditions of their degeneration into a solitary wave). It is therefore natural to adopt the limitations set forth by Lavrent'ev [1]

$$f' = O(\varepsilon^{3/2}), \qquad f'' = O(\varepsilon^2), \qquad f''' = O(\varepsilon^{5/2}) \quad \text{etc.} \tag{1.5}$$

which are valid for solitary waves in an ideal fluid.

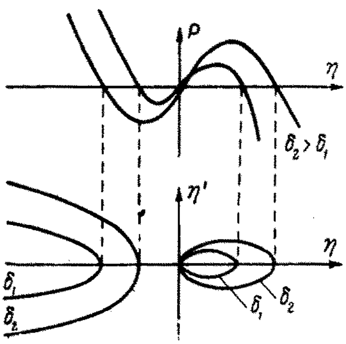

Fig. 2

Under conditions (1.5) and (1.4) we have an asymptotic representation for the velocity modulus on the curve $y = f(x)$,

$$(v_x^2 + v_y^2)_{y=f(x)} = \frac{1}{f^2}\left[1 + \frac{2}{3} ff''\right] + O(\varepsilon^{5/2}) \tag{1.6}$$

Discarding terms of the order we can write Equation (1.2) as $O(\varepsilon^{5/2})$,

$$f''\left[1 - \frac{3}{2}\gamma f\right] + 3\gamma f^2 + \frac{3}{2f} - \frac{3}{2} cf = 0 \tag{1.7}$$

Equation (1.7) has a first integral, and its phase plane can be investigated easily. It is that portion of it on which conditions (1.5) are fulfilled, however, that is germane to the discussion below. These conditions are necessarily fulfilled for $f = 1$. It is therefore possible to obtain the same qualitative results without restoring to cumbersome computation by setting $f = 1 + \eta$ and retaining the first few terms of the expansions in (1.7). Limiting ourselves to terms of the order $O(\eta^2)$, we reduce (1.7) to the form

$$(1 - {}^3/_2\gamma)\,\eta'' + A\eta + B\eta^2 = D \tag{1.8}$$

$$\begin{aligned} A &= 3\nu(2 + \beta) + {}^3/_2(\beta - 1) - {}^3/_2 C(1 + \beta) \\ B &= 3\nu(1 + 2\beta + \beta^2) + {}^3/_2(1 - \beta + \beta^2) - {}^3/_2 C(\beta + \beta^2) \\ D &= {}^3/_2 C - 3\nu - {}^3/_2 \qquad (\beta = {}^3/_2\gamma / (1 - {}^3/_2\gamma)) \end{aligned} \tag{1.9}$$

To simplify notations, we replace C by the new constant δ: $C = 2\gamma + 1 + \delta$. Formulas (1.9) then become

$$\begin{aligned} A &= 3(\nu - 1) - {}^3/_2\delta(1 + \beta) \\ B &= 3\nu + {}^3/_2 + 3(\nu - 1)\beta - {}^3/_2\delta\beta(1 - \beta), \qquad D = {}^3/_2\delta \end{aligned} \tag{1.10}$$

Equation (1.8) admits of the first integral

$${}^1/_2(1 - {}^3/_2\gamma)\,\eta'^2 = F + D\eta - {}^1/_2 A\eta^2 - {}^1/_3 B\eta^3 \equiv P(\eta) \tag{1.11}$$

where F is a new constant which we can assume to be zero. This is equivalent to saying that instead of a linear dimension we choose the depth of fluid at the peak of the hump or at the trough of the wave. The value $\eta = 0$ then corresponds to $\eta' = 0$. Thus, the behavior of the phase curves is determined by the three parameters ν, δ and γ.

3°. First let us consider the case where $\delta \geqslant 0$, $\gamma < {}^2/_3$ and $B \geqslant 0$. The latter condition is fulfilled, for example, if ν is close to unity and δ is small. Fig.2 shows the curves $P(\eta)$ and the phase plane of Equation (1.8) for this case. The curve branches in the left half-plane have no physical meaning, since they describe the unbounded solutions of Equation (1.8). The branches lying in the right half-plane describe periodic solutions whose amplitude is all the smaller, the smaller the value of δ. Thus, when $\delta > 0$, waves propagate along the surface whose length λ is given by the quadrature

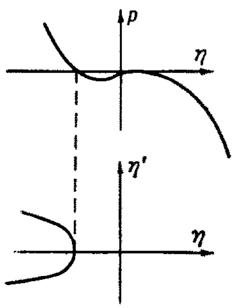

Fig. 3

$$\lambda = \sqrt{2}\ \frac{d\eta}{\sqrt{(D\eta - {}^1/_2 A\eta^2 - {}^1/_8 B\eta^3)(1 - {}^3/_2\gamma)}} \qquad (1.12)$$

in which the integral extends over the entire phase trajectory. From (1.12) we see that $\lim \lambda = \infty$ for $\delta \to 0$. If $\nu > 1$ (i.e. if the velocity of propagation of the wave relative to the stationary fluid $V < \sqrt{gh}$), then for $\delta = 0$ the phase diagram of the flow is as shown in Fig.3.

Thus, the periodic solution in this case degenerates to the point $\eta = \eta' = 0$, i.e. the flow degenerates into a plane-parallel stream. An entirely different situation obtains when $\nu < 1$. The phase diagram of flow in this case is shown in Fig.4. We see that as $\lambda \to \infty$ the maximum elevation ("amplitude") η^+ does not tend to zero. The free surface in this case constitutes a solitary wave.

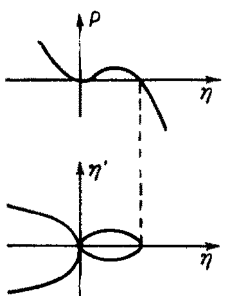

Fig. 4

Thus, if $\nu > 1$, then as $\lambda \to \infty$ the waves which propagate with a given velocity (ν is fixed) degenerate to a uniform stream. The amplitude of these waves $\eta^* \to 0$, they were studied by Litman [3 for $\gamma = 0$. If $\nu < 1$, then $\lim \eta^* > 0$ as $\lambda \to \infty$, and the limiting solution constitutes a solitary wave which has the least amplitude $\eta^*(\nu)$ of all waves propagating with a given velocity; furthermore, the amplitude of the solitary wave tends to zero as $\nu \to 1$. These waves have been studied by Kortweg and de Vries [4; the corresponding existence theorem has been proved by Sekerzh-Zenkovich [5]. The value $\nu = 1$ is bifurcational. For $\nu > 1$ there exists a unique form of motion of the infinitely long wave, to wit, a uniform stream. The two limiting forms (a uniform stream and a solitary wave) are realized with $\nu < 1$. The waves under investigation here are similar to ordinary gravitational (conoid) waves into which they evolve as $\gamma \to 0$. Cases of such waves can be easily studied by analytic methods. For $\nu > 1$ the wave motions can be studied by quasilinear methods (the Liapunov-Poincaré or Krylov-Bogoliubov methods), since even in the first approximation, Equation (1.8) already admits of the periodic solution

$$\eta = \eta^* \cos\left[\left(\frac{3(\nu-1)}{1-{}^3/_2\gamma}\, x\right)^{1/2} + \theta\right] \qquad (\theta = \mathrm{const}) \qquad (1.13)$$

If $\nu < 1$, then the equation of the first approximation does not contain periodic solutions, so that such waves cannot be studied in linear approximation (*). In this case the solution can be obtained in elliptic functions. The solution becomes especially simple for $\delta = 0$. It is easy to verify that in this case Equation (1.8) has the solution

$$\eta = \frac{3\,|A|}{2B\cosh^2 {}^1/_2 \sqrt{|A|}\, x} = \frac{\eta^*}{\cosh^2 {}^1/_2 \sqrt{A}\, x} \qquad (1.14)$$

which tends to zero as $|x| \to \infty$.

Hence we see that the amplitude of the solitary wave

$$\eta^* = \frac{3(1-\nu)}{2\nu + 1 - 3(1-\nu)\beta}$$

increases with increasing surface tension.

4°. It is easy to verify that the case $\delta < 0$ has no physical meaning.

In fact, let us consider the asymptotic theory assuming in advance that the solution satisfies conditions (1.5). It follows from (1.13) and (1.14) that the above prior estimates are valid if ν is close to unity. Thus, as ϵ we must adopt the quantity $\sqrt{|1-\nu|}$, and the above investigation is meaningful only in the neighborhood of the point $\nu = 1$.

5°. We have already considered the case of γ restricted to small values. The surface tension forces merely alter the wave parameters but do not produce new forms of motion. Let us assume now that $\gamma > {}^2/_3$. Upon passage of the parameter γ through the bifurcational value $\gamma = {}^2/_3$, there occurs a change in the signs of the coefficients A, B and D in Equation (1.8). Investigating the phase as was done above, we arrive at the following results.

a) If $\gamma > {}^2/_3$ and $\nu < 1$, then as $\lambda \to \infty$ the waves degenerate into a uniform stream, i.e. the wave amplitude vanishes as $\lambda \to \infty$.

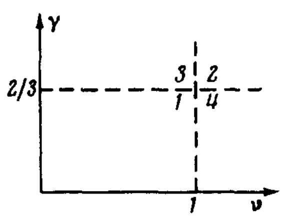

Fig. 5

b) If $\gamma > {}^2/_3$ and $\nu > 1$, then as $\lambda \to \infty$ the amplitude has a finite limit, and this limiting flow constitutes a solitary wave. These waves are trough- and not hump-shaped, however.

Thus, if the parameters γ and ν belong to 3 and 4 (Fig.5), then as $\lambda \to \infty$ there exists a unique limiting form of motion, that is a uniform stream. If the parameters γ and ν belong

*) In strict (nonasymptotic) formulation the wave problem can be reduced to an operator equation with an operator whose Fréchet derivative vanishes for $\nu = 1$ and $\delta = 0$.

to the regions 1 and 2, then as $\lambda \to \infty$ there exist two limiting forms of motion: in addition to a uniform stream there is a motion wherein the free surface is wave-shaped, the solitary wave having the shape of a hump for parameter values belonging to region 1 and a trough shape for region 2.

N o t e . The above asymptotic analysis is meaningful only if

$$|A|\cdot|1 - {}^3/_2\gamma|^{-1}$$

is a small quantity. We have considered the case when the smallness of this quantity is assured by the closeness of ν to 1 and δ to zero. However, this quantity can be small even if the denominator is large. This yields a new type of fluid motion which can be studied by asymptotic methods for any value of ν. For example, let $\nu = 0$, i.e. let us consider purely capillary waves.

Then

$$\frac{A}{1 - {}^3/_2\gamma} = -\frac{2\delta}{3(1 - {}^2/_3\gamma)^2}$$

Equation (1.8) admits of a solution of the solitary wave type provided that $D = 0$, i.e. that $\delta = 0$. But in this instance $A = 0$ as well, although here ($\gamma > {}^2/_3$) Equation (1.9) simply has no bounded solutions with the exception of the trivial, i.e. purely capillary flows do not contain a solitary wave.

2. Axisymmetrical jets. 1°. Let us consider an axisymmetrical jet acted upon by surface tension forces; the flow is assumed to be steady. The purpose of our investigation is to study the possible forms of the jet. The equation of the jet boundary will be specified in the form $r = f(z)$. The jet flow is assumed to be potential. Inasmuch as the jet is assumed axisymmetrical, the problem may be reduced to finding the stream function $\psi(r, z)$ satisfying Equation

$$\psi_{rr} + \psi_{zz} - r^{-1}\psi_r = 0 \tag{2.1}$$

and the function $f(z)$ in accordance with the following conditions:

$$\psi = 1 \quad \text{for } r = f(z), \qquad \psi = 0 \quad \text{for } r = 0 \tag{2.2}$$

$$f^{-2}[\psi_r^2 + \psi_z^2] - 2\gamma K = C \quad \text{for } r = f(z) \tag{2.3}$$

Condition (2.2) is a kinematic condition (in this case the condition of a constant flow rate), and (2.3) is a dynamic condition. The problem is formulated in terms of dimensionless variables. As the characteristic parameters we have chosen the radius R of the jet at some cross section (whose position will be established below) and the value Q of the stream function on the jet surface. C is the constant energy, which is a functional, and K is the mean curvature of the jet surface,

$$K = \frac{1}{2}\left(\frac{1}{f\cos\theta} + \frac{f''}{(1+f'^2)^{3/2}}\right) = \frac{1}{2}\left[\frac{(1+f'^2)^{1/2}}{f} + \frac{f''}{(1+f'^2)^{3/2}}\right] \tag{2.4}$$

where γ is a dimensionless parameter characterizing the action of surface tension forces,

$$\gamma = \frac{\alpha}{RV^2}, \quad \text{or} \quad \gamma = \frac{\alpha R^3}{Q^2} \qquad \left(V = \frac{Q}{R^2}\right) \tag{2.5}$$

Thus we see that the role of surface tension is in direct proportion to the coefficient of surface tension α and inversely proportional to the jet thickness and characteristic velocity of the fluid particles.

2°. The function ψ satisfying (2.1) within the strip $0 < r < f(z)$, which becomes unity on the boundary $r = f(z)$ and zero on the straight line $r = 0$, has the following asymptotic representation (see [1], p.185):

$$\psi \sim \frac{r^2}{f^2} + \frac{3f'^2 - f''f}{4f^2} r^2 \left(1 - \frac{r^2}{f^2}\right) + \cdots \qquad (2.6)$$

and similarly

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial z} \sim \frac{2rf'}{f^3} + \cdots, \quad v_z = \frac{1}{r}\frac{\partial \psi}{\partial r} \sim -\left[\frac{2}{f^2} + \frac{3f'^2 - f''f}{2f^2}\left(1 - \frac{2r^2}{f^2}\right) + \cdots\right] \qquad (2.7)$$

furthermore,

$$v_r^2 + v_z^2 = \frac{1}{f^2}(\psi_r^2 + \psi_z^2) \sim \frac{4r^2f'^2}{f^6} + \frac{4}{f^4} + \frac{6f'^2 - 2ff''}{f^4}\left(1 - \frac{2r^2}{f^2}\right) + \cdots \qquad (2.8)$$

Since the boundary is unknown, we must make use of some prior estimates of it. With the same aims as those set forth in the preceding Section, we once again employ the estimates of Lavrent'ev,

$$f' = O(\varepsilon^{5/2}), \quad f'' = O(\varepsilon^2) \ \text{etc.} \qquad (2.9)$$

The validity of these estimates and the meaning of the parameter ε will be established below. Making use of estimates (2.9), we rewrite Formulas (2.7) and (2.8) in the form

$$\psi = \frac{r^2}{f^2} - \frac{f''}{4f^2} r^2 \left(1 - \frac{r^2}{f^2}\right) + O(\varepsilon^{5/2})$$
$$(v_r^2 + v_z^2)_{r=f} = \frac{4}{f^4}\left[1 + \frac{1}{2} ff''\right] + O(\varepsilon^{5/2}) \qquad (2.10)$$

In accordance with estimates (2.9), we transform (2.4),

$$K = {}^1/_2 (f^{-1} + f'') + O(\varepsilon^3) \qquad (2.11)$$

Substituting (2.10) and (2.11) into dynamic condition (2.3), we arrive at an ordinary second-order differential equation in the function f (i.e. the free boundary),

$$(2 - \gamma f^3) f'' + 4f^{-1} - \gamma f^2 - Cf^3 = 0 \qquad (2.12)$$

We note that Equation (2.12) admits of a first integral whose properties can easily be investigated by phase plane methods.

For our purposes it is sufficient to consider the quadratic approximation. To this end we set $f = 1 + \eta$ and retain terms on the order of $O(\eta^2)$ in (2.12).

Equation (2.12) then becomes

$$(2 - \gamma)\eta'' + A\eta + B\eta + B\eta^2 = D \qquad (2.13)$$

where the constants A, B and D are given by Formulas

$$A = -\beta D - 4 - 2\gamma - 3C, \quad D = \gamma + C - 4$$
$$B = -D - 2C - \beta(3\gamma - 4C) - \beta^2 D \quad (\beta = 3\gamma/(2 - \gamma)) \qquad (2.14)$$

3°. In the case of a uniform stream and solitary wave we have $\eta'' = 0$ and $\eta = 0$ as $|z| \to \infty$. Hence, in this case $D = 0$ and $C = 4 - \gamma$. Let us consider flows closely resembling the latter. We set $C = 4 - \gamma + \delta$ and assume $D = \delta$ to be small. In this case

$$A = \gamma - 16 - \delta (3 + \beta)$$

Since the theory being presented is valid for small A, we set $\gamma = 16 + \Delta$, whereupon $\beta = -{}^{24}/_7 + {}^3/_{98} + O(\Delta^2)$; moreover,

$$A = {}^3/_7\delta + \Delta + O(\max(\delta^2, \Delta^2)), \qquad B = 13\,{}^2/_7 + O(\max(|\Delta|, |\delta|))$$

Thus, for small Δ and δ we have $b > 0$, and the sign of A is determined by the relationship between Δ and δ. Noting furthermore that $2 - \gamma < 0$ for small Δ and reasoning as in the preceding Section, we can easily obtain the following results.

a) If $\delta < 0$, i.e. if the constant energy $C < -12 + \Delta$, then Equation (2.13) has no bounded solutions. This means that a jet extending to infinity to either side expands without limit in at least one of the directions.

b) If $\delta \geqslant 0$, then the Equation (2.13) admits of solutions bounded on the entire straight line $-\infty < z < +\infty$. If $\delta \neq 0$, then these solutions constitute nonlinear waves whose amplitudes decrease along with δ.

c) The character of limiting solutions as $\lambda \to \infty$ (i.e. as $\delta \to 0$) depends to a marked extent on Δ. If $\Delta > 0$ (i.e. $\gamma > 16$), then

$$\lim \frac{A}{2-\gamma} < 0 \quad \text{for } \delta \to 0$$

and the amplitude of the limiting solution is not equal to zero. The limiting solution constitutes a solitary wave of the trough type. If $\Delta < 0$, then the limiting solution constitutes a uniform stream.

3. Nonstationary problems of jet theory. 1°. Let us construct the equations of long axisymmetrical waves propagating along a cylindrical jet of an ideal weightless fluid acted upon by capillarity forces. The velocity potential φ and the shape of the free surface $r = f(z, t)$ is defined as the solution of the following problem (Fig.6):

$$\frac{\partial}{\partial r}\left(r\frac{\partial\varphi}{\partial r}\right) + \frac{\partial}{\partial z}\left(r\frac{\partial\varphi}{\partial z}\right) = 0, \qquad V_r = \frac{\partial\varphi}{\partial r} = 0 \quad \text{for } r = 0 \tag{3.1}$$

$$\frac{\partial\varphi}{\partial t} = \frac{\partial\varphi}{\partial r} - \frac{\partial f}{\partial z}\frac{\partial\varphi}{\partial z} \quad \text{for } r = f \tag{3.2}$$

$$\frac{\partial\varphi}{\partial t} + \frac{1}{2}\left(\frac{\partial\varphi}{\partial r}\right)^2 + \frac{1}{2}\left(\frac{\partial\varphi}{\partial z}\right)^2 - 2\gamma K = C \quad \text{for } r = f \tag{3.3}$$

The dimensionless variables are introduced as in the preceding Section.

2°. First let us consider an auxiliary problem on the determination of the velocity potential which satisfies Equation (3.1), condition (3.2), and condition

$$\varphi(f, z, t) = a(z, t) \tag{3.4}$$

Using ϵ to denote a small parameter, we carry out the substitution of variable $z = \xi/\epsilon$ in (3.1).

Equation (3.1) then becomes

$$\frac{\partial}{\partial r}\left(r\frac{\partial\varphi}{\partial r}\right)+\varepsilon^2\frac{\partial}{\partial\xi}\left(r\frac{\partial\varphi}{\partial\xi}\right)=0 \tag{3.5}$$

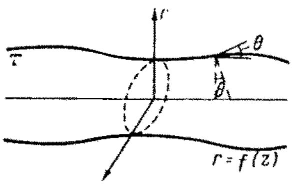

Fig. 6

We shall attempt to find a solution of (3.5) in the form of a series

$$\varphi=\varphi_0+\varepsilon^2\varphi_1+\ldots \tag{3.6}$$

where φ_i satisfy Equations

$$\frac{\partial}{\partial r}\left(r\frac{\partial\varphi_0}{\partial r}\right)=0,\quad \frac{\partial}{\partial r}\left(r\frac{\partial\varphi_1}{\partial r}\right)=-r\frac{\partial^2\varphi_0}{d\xi^2},\ldots \tag{3.7}$$

The functions φ_i satisfy the following boundary conditions:

$$\partial\varphi_i/\partial r=0 \quad \text{for } r=0,\qquad \varphi_0=a \quad \text{for } r=f,\qquad \varphi_i=0\quad (i>0) \quad \text{for } r=f$$

Solving this sequence of boundary-value problems and converting back to the variable z, we obtain the following expression for s in terms of its boundary values:

$$\varphi\sim a(z,t)+{}^1/_4 a_{zz}(f^2-r^2)+\ldots \tag{3.8}$$

In order for (3.9) to have meaning, it is necessary and sufficient that the derivatives of the potential a and of the function f decrease sufficiently rapidly as their order increases. This condition is fulfilled if the waves are long enough. Using representation (3.8) for the potential, we can transform the kinematic and dynamic relations (3.2) and (3.3),

$$\frac{\partial f}{\partial t}=\frac{1}{4}a_{zz}f-f_za_z+\ldots,\ \frac{\partial a}{\partial t}+\frac{1}{2}a_z^2-\gamma\left(\frac{1}{f}+f_{zz}\right)+\ldots=C \tag{3.9}$$

By limiting ourselves to a specific number of terms in Equations (3.9), we obtain equations giving an approximate description of the propagation of axisymmetrical nonlinear waves along the jet.

3°. Equations (3.9) can be used to investigate various phenomena arising in jet theory. Specifically, it is possible to investigate the stability of jets with respect to long perturbations. Plateau [6] was apparently the first to note that surface tension forces may be the cause of jet instability. A similar study of this matter was performed by Reley [7], who showed that the cylindrical shape of a jet is unstable with respect to sufficiently long perturbations. This fact follows from system (3.9).

We set $a=V_z+b$ and $f=1+\eta$ and linearize system (3.9), retaining in it all derivatives up to the second order,

$$\eta_t={}^1/_4 b_{zz}-V\eta_z,\qquad b_t+2b_zV-\gamma(-\eta+\eta_{zz})=0 \tag{3.10}$$

Let us attempt to find a solution of the running wave type. To do this we set

$$\eta=Ae^{i\omega z+\mu t},\qquad b=Be^{i\omega z+\mu t} \tag{3.11}$$

For determining A and B we have Equations

$$A(\mu+iV\omega)+{}^1/_4\omega^2B=0,\qquad A\gamma(1+\omega^2)+B(\mu+2i\omega V)=0 \tag{3.12}$$

We note that if $\gamma=0$, i.e. if there is no surface tension, the second equation of (3.12) immediately yields $\mu=-2i\omega V$, i.e. the jet is in neutral equilibrium in the absence of surface tension forces. If surface tension forces are added, the jet becomes unstable. In fact, the characteristic equation of system (3.12) is of the form

$$\mu^2+3i\omega V\mu+\Delta \qquad (\Delta=-{}^1/_4\omega^2\gamma(1+\omega^2)<0) \tag{3.13}$$

This immediately implies that one of the roots of (3.14) has a positive real part. Thus, a thin jet is disrupted by surface tension forces. Integrating system (3.9), we can trace the process of disintegration of the jet into drops. The above theory is valid only for long perturbations. The trivial form of a jet of weightless fluid acted upon by surface tension forces is therefore unstable. However, as we see from the preceding Section, other forms of axisymmetrical jets are possible. In particular, these may be wave-shaped. This naturally leads us to ask whether stable flows might exist among the possible forms of jet flows.

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